From subcritical behavior to a correlation-induced transition in rumor models

Guilherme Ferraz de Arruda,[1,*] Lucas G. S. Jeub,[2] Angélica S. Mata,[3] Francisco A. Rodrigues,[4] and Yamir Moreno[2,5,6]

[1] *CENTAI Institute, Turin 10138, Italy*

[2] *ISI Foundation, Via Chisola 5, 10126 Torino, Italy*

[3] *Departamento de Física, Universidade Federal de Lavras, 37200-900, Lavras, Minas Gerais, Brazil*

[4] *Departamento de Matemática Aplicada e Estatística, Instituto de Ciências Matemáticas e de Computação, Universidade de São Paulo - Campus de São Carlos, Caixa Postal 668, 13560-970 São Carlos, SP, Brazil.*

[5] *Institute for Biocomputation and Physics of Complex Systems (BIFI), University of Zaragoza, Zaragoza 50009, Spain*

[6] *Department of Theoretical Physics, University of Zaragoza, Zaragoza 50009, Spain*

Rumor and information spreading are natural processes that emerge from human-to-human interaction. Such processes have a growing impact on people's daily lives due to increasing and faster access to information, whether trusted or not. A popular mathematical model for spreading rumors, data, or news is the Maki–Thompson (MT) model [1]. In this model, individuals can be in one of three states: ignorant, spreader, or stifler. The spreading evolves through the contact between nodes defined by an undirected network. Our process is defined in continuous time as a collection of Poisson processes. If the contact is between a spreader and an ignorant, the second node will learn the rumor and become another spreader at rate λ. On the other hand, if the contact happens between a spreader and someone that already knows the rumor (spreader or stifler), then the spreader that initiated the contact will lose interest in the rumor, thus becoming a stifler at a rate α. Existing work based on first-order mean-field approximations suggested that this model does not have a phase transition with rumors always reaching a finite fraction of the population irrespective of the spreading rate [2–4]. Here, we show that a second-order phase transition is present in this model, which is not captured by first-order mean-field approximations. Since the MT model has infinitely many absorbing states, the critical point is the spreading parameter that separates the two scaling regimes. Before this point, the final number of stiflers when the process reaches an absorbing state does not scale with the system size, and hence its fraction goes to zero in the thermodynamic limit. After the critical point, the number of stiflers scale with the system size. This transition is shown in Fig. 1 (a) and (b), where we present the order parameter (the fraction of stiflers) and the time to reach the absorbing state, respectively. Moreover, we propose and explore a modified version of the Maki–Thompson model that includes a forgetting mechanism, where each stifler spontaneously becomes ignorant at a rate δ. This modification changes the Markov chain's nature from infinitely many absorbing states in the classical setup to a single absorbing state and allows us to use a plethora of analytic and numeric methods to characterize the model's behavior. In particular, we were able to provide an estimation of the critical point by accounting for the correlations between states. The accuracy of our approximation is shown in Fig. 1 (c). More importantly, we find a counter-intuitive behavior in the subcritical regime, where the lifespan of a rumor increases as the spreading rate drops, following a power-law relationship. These results are summarized in Fig. 1 (b) and (d). Specifically, in Fig. 1 (b), we present the time to reach the absorbing state for different sizes, demonstrating the power-law subcritical behavior. Complementary, in Fig. 1 (d), we compare a similar result with two analytical approximations, confirming the power-law subcritical behavior. This behavior implies that, even below the critical threshold, rumors can survive for a long time. Furthermore, using an asymptotic analysis where we scale the model's parameters, we were able to show that no phase transition is expected in the first-order mean field approximation. This result, together with our critical point estimations, emphasizes the role of correlations in the MT model phase transition and motivates further research on developing more sophisticated mean-field approximations. Together, our findings are at odds with most classical results and show that the dynamic behavior of rumor models is much richer than previously thought. Thus, we hope our results motivate further analytical and numerical research and investigations involving real-world systems. The work described in this abstract has been published in [5].

* gui.f.arruda@gmail.com

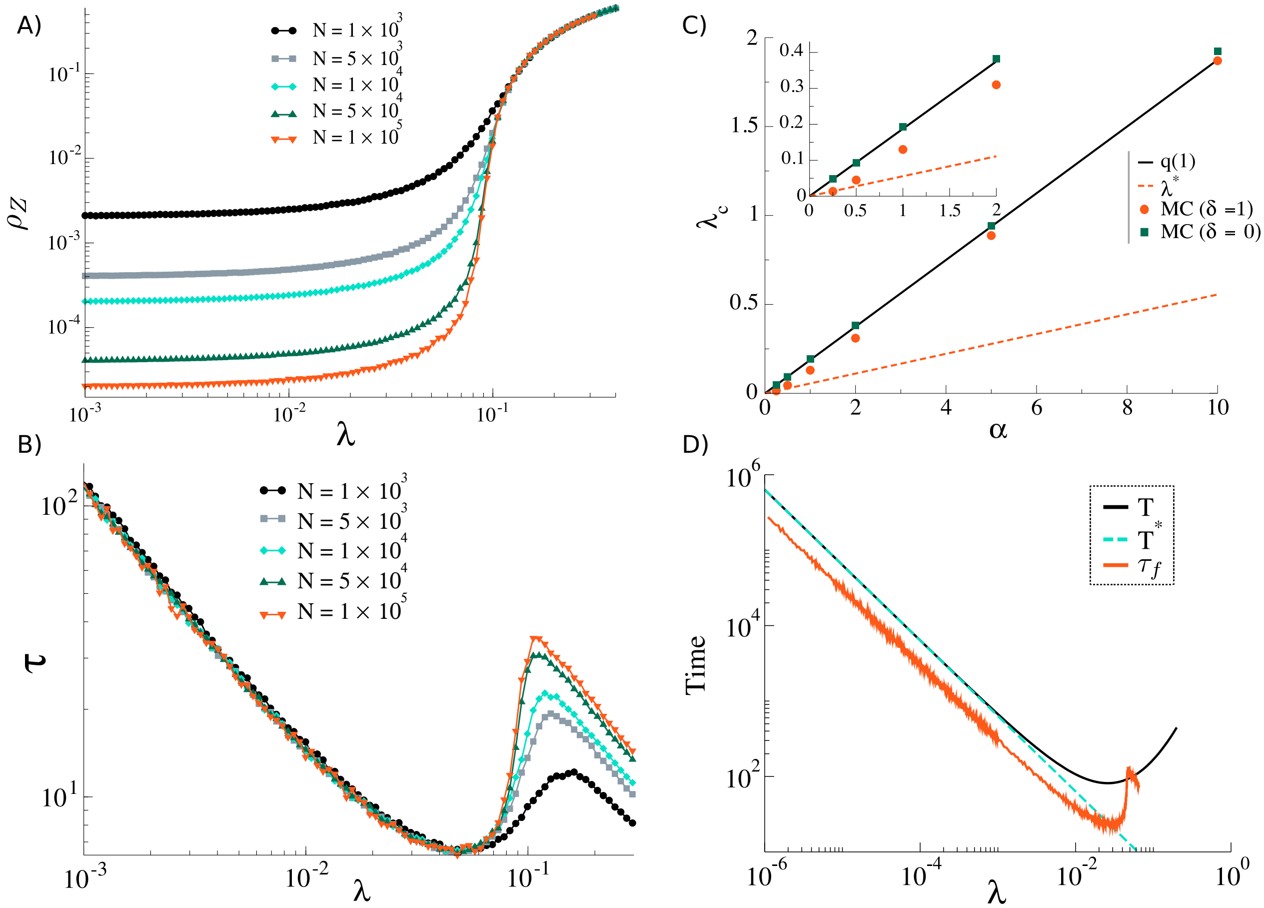

FIG. 1. **Summary of results showing the phase transition in the Maki–Thompson model.** In (a) and (b), we show the phase diagram and the time to reach the absorbing state for the standard MT model, $\alpha = 1$, and different sizes on a random regular network with $\langle k \rangle = 10$, respectively. In (c), we present the comparison between analytical and Monte Carlo critical point estimations for random regular networks with $\langle k \rangle = 10$ and $\delta = 1$ and $N = 10^6$. In the inset, we present the comparison for the low α regime. Finally, in (d), we show two approximations for the time to reach the absorbing state, T and T^*, as a function of λ for $\delta = 1.0$ and $\alpha = 0.5$. In this subfigure, the red curve results from Monte Carlo simulations in a random regular network with the same parameters, $\langle k \rangle = 10$ and $N = 10^5$.

[1] D. P. Maki and M. Thompson, *Mathematical models and applications* (Prentice-Hall Inc., Englewood Cliffs, N.J., 1973).
[2] Y. Moreno, M. Nekovee, and A. F. Pacheco, Phys. Rev. E **69**, 066130 (2004).
[3] M. Nekovee, Y. Moreno, G. Bianconi, and M. Marsili, Physica A: Statistical Mechanics and its Applications **374**, 457 (2007).
[4] A. Barrat, M. Barthlemy, and A. Vespignani, *Dynamical processes on complex networks* (Cambridge University Press New York, NY, USA, 2008).
[5] G. Ferraz de Arruda, L. G. S. Jeub, A. S. Mata, F. A. Rodrigues, and Y. Moreno, Nature Communications **13**, 3049 (2022).